Scattering of surface waves with floating bridge in presence of floating horizontal porous plate over trench type bottom topography

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Abstract: The problem involving oblique wave interaction with a floating bridge in the presence of a floating horizontal porous plate over a trench-type bottom is investigated. The role of the porous plate and the trench is analyzed in detail to reduce wave forces on the bridge. Significant changes are found in forces due to this porous breakwater and trench type of bottom topography compared to the case without breakwater and trench. In addition, the maxima in wave energy dissipation are associated with the minima in wave forces acting on the floating bridge. The findings from the present model are likely to be helpful in understanding the role of porous breakwater and trench in engineering applications.
Keywords: Floating bridge, Horizontal porous plate, Trench, Eigenfunction expansion, Wave forces.

1 Introduction

In recent decades, ocean space has been targeted as one of the alternatives to meet the growing demand for infrastructures in the form of floating bridges, airports, military bases and entertainment facilities. To increase the life-span of the floating object or floating bridge, there is a need to investigate various means to mitigate the wave-induced forces acting on floating bridges. Near these infrastructures, the construction of a breakwater such as a porous barrier (see [1-9]) leads to a significant reduction in wave-induced forces. Most of the above investigations involve interaction of waves with one or two barriers over uniform finite depth of water. Since the bottom of the ocean is hardly non-conflicting throughout, it is justifiable to study the propagation of surface water waves over a non-uniform bottom topography due to their significant usage in marine and coastal engineering. In the last few decades, problems of scattering of water waves involving uneven bottom topography have been investigated to find out the characteristics of the conservation of wave energy by many researchers (see [10-13]).

Hence, the present paper develops a model by placing the horizontal porous plate over the asymmetric trench type of bottom topography and at a finite distance from a floating bridge. The effectiveness of the horizontal porous breakwater is studied. The effects of various physical parameters of porous barriers and bridges are analyzed to mitigate the wave forces acting on the bridge and to protect the bridge.

2 Mathematical formulation

It is assumed that the fluid is inviscid, incompressible and motion is irrotational and simple harmonic in time. Using the linear theory, the spatial velocity potential ϕ_j in each region j ($j = 1, 2...7$), which satisfies the governing equation

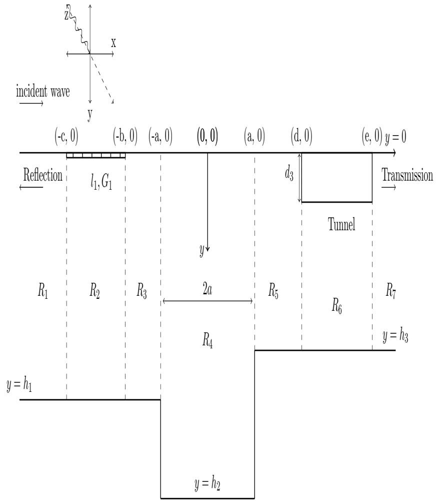
$$\frac{\partial^2 \phi_j}{\partial^2 x} + \frac{\partial^2 \phi_j}{\partial^2 y} - l^2 \phi_j = 0; j = 1, 2...7 \tag{1}$$

with boundary conditions:

$$\frac{\partial \phi_j}{\partial y} + (\omega^2/g)\phi_j = 0 \qquad \text{on } y = 0,\ j = 1,3,4,5,7 \tag{2}$$

$$\phi_{2y}(x,0) = -ik_0 G\phi_2(x,0) \qquad \text{on -}c \le x \le -b. \tag{3}$$

$$\left.\begin{aligned} \frac{\partial \phi_j}{\partial y} &= 0,\ \text{on } y = h_1,\ j = 1,2,3 \\ \frac{\partial \phi_j}{\partial y} &= 0,\ \text{on } y = h_2\ j = 4 \\ \frac{\partial \phi_j}{\partial y} &= 0,\ \text{on } y = h_3\ j = 5,6,7 \end{aligned}\right\} \tag{4}$$



Condition on the trench walls:

$$\left.\begin{aligned} &\phi_{5x}(-a,y) = 0,\ \text{for } h_1 < y < h_2, \\ &\phi_{5x}(a,y) = 0,\ \text{for } h_3 < y < h_2, \end{aligned}\right\} \tag{5}$$

Condition on vertical and horizontal walls of floating bridge:

$$\left.\begin{aligned} \frac{\partial \phi_5}{\partial x} &= 0, \quad \text{on } x = d \text{ and } 0 \le y \le d_3, \\ \frac{\partial \phi_7}{\partial x} &= 0 \quad \text{on } x = e \text{ and } 0 \le y \le d_3, \end{aligned}\right\} \tag{6}$$

$$\frac{\partial \phi_6}{\partial y} = 0 \quad \text{on } y = d_3 \text{ and } d \le x \le e, \tag{7}$$

Matching the continuity of velocity and pressure along the mutual boundaries at $x = -c,\ x = -b,\ x = -a,\ x = a,\ x = d,$ and $x = e$ then yields

$$\left.\begin{aligned} &\phi_1(-c,y)\& = \phi_2(-c,y), && 0 < y < h_1, \\ &\phi_{1x}(-c,y)\& = \phi_{2x}(-c,y), && 0 < y < h_1, \end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned} &\phi_2(-b,y) = \phi_3(-b,y), && 0 < y < h_1, \\ &\phi_{2x}(-b,y) = \phi_{3x}(-b,y), && 0 < y < h_1, \end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned} &\phi_3(-a,y) = \phi_4(-a,y), && 0 < y < h_1, \\ &\phi_{3x}(-a,y) = \phi_{4x}(-a,y), && 0 < y < h_1, \end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned} &\phi_4(a,y) = \phi_5(a,y), && 0 < y < h_3, \\ &\phi_{4x}(a,y) = \phi_{5x}(a,y), && 0 < y < h_3, \end{aligned}\right\} \tag{11}$$

$$\left.\begin{aligned} &\phi_5(d,y) = \phi_6(d,y), && d_3 < y < h_3, \\ &\phi_{5x}(d,y) = \phi_{6x}(d,y), && d_3 < y < h_3, \end{aligned}\right\} \tag{12}$$

$$\left.\begin{aligned} &\phi_6(e,y) = \phi_7(e,y), && d_3 < y < h_3, \\ &\phi_{6x}(e,y) = \phi_{7x}(e,y), && d_3 < y < h_3, \end{aligned}\right\} \tag{13}$$

condition on the trench walls:

$$\left.\begin{aligned} &\phi_{5x}(-a,y) = 0,\ \text{for } h_1 < y < h_2, \\ &\phi_{5x}(a,y) = 0,\ \text{for } h_3 < y < h_2. \end{aligned}\right\} \tag{14}$$

The far-field behaviour given by

$$\left.\begin{aligned} &\phi_1 \to (\frac{ig}{\omega})\frac{\cosh k_0(h_1 - y)}{\cosh k_0 h_1}\{e^{iK_0(x+c)} + R_0 e^{-iK_0(x+c)}\},\ \text{for as } x \to -\infty \\ &\phi_7 \to (\frac{ig}{\omega})\frac{\cosh p_0(h_3 - y)}{\cosh p_0 h_3}\{T_0 e^{iP_0(x-e)}\},\ \text{for as } x \to \infty. \end{aligned}\right\} \tag{15}$$

where, ω is angular frequency, g is acceleration due to gravity, G is the porous effect parameters of the porous plate, θ_1 is the angle of incidence. $K_0 = k_0 \cos\theta_1$ and k_0 is the wave number of plane progressive waves and it is the real root the transcendental equation

$$k \tanh(kh_1) = \omega^2/g, \tag{16}$$

and $P_0 = \sqrt{{p_0}^2 - (k_0 \sin\theta_1)^2}$ and p_0 is positive the real root of transcendental equation

$$p \tanh(ph_3) = \omega^2/g. \tag{17}$$

R_0 and T_0 are unknowns associated with the amplitude of the reflected and transmitted waves respectively to be determined here.

3 Method of solution

To find ϕ_j, $j = 1, 2, ..., 7$, a Havelock's expansion (ref.[14]) of $\phi_j(x, y)$ is written in terms of eigenfunctions in each region and we substitute in the matching conditions (7) to (13), we get

$$R_m N_m - \sum_{n=0}^{\infty} A_{mn} C_n - \sum_{n=0}^{\infty} A_{mn} e^{iQ_n l_1} D_n = -\epsilon_m; m = 0, 1, 2, ..., \tag{18a}$$

$$K_m R_m N_m + \sum_{n=0}^{\infty} Q_n A_{mn} C_n - \sum_{n=0}^{\infty} Q_n A_{mn} e^{iQ_n l_1} D_n = K_0 \epsilon_m; m = 0, 1, 2, ..., \tag{18b}$$

$$\sum_{n=0}^{\infty} e^{iQ_n l_1} A_{mn} C_n + \sum_{n=0}^{\infty} A_{mn} D_n - N_m E_m - N_m e^{iK_m l_2} F_m = 0; m = 0, 1, 2, ..., \tag{18c}$$

$$-\sum_{n=0}^{\infty} Q_n e^{iQ_n l_1} A_{mn} C_n + \sum_{n=0}^{\infty} Q_n A_{mn} D_n + K_m N_m E_m - K_m N_m e^{iK_m l_2} F_m = 0; m = 0, 1, 2, ..., \tag{18d}$$

$$(N_m e^{iK_m l_2}) E_m + N_m F_m - \sum_{n=0}^{\infty} B_{mn} A_n - \sum_{n=0}^{\infty} B_{mn} e^{i\hat{K}_n l_3} B_n = 0; m = 0, 1, 2, ..., \tag{18e}$$

$$(N_m K_m e^{iK_m l_2}) E_m - (K_m N_m) F_m - \sum_{n=0}^{\infty} (\hat{K}_n B_{mn}) A_n + \sum_{n=0}^{\infty} (B_{mn} \hat{K}_n e^{i\hat{K}_n l_3}) B_n = 0; m = 0, 1, 2, ..., \tag{18f}$$

$$\sum_{n=0}^{\infty} (C_{mn} e^{i\hat{K}_n l_3}) A_n + \sum_{n=0}^{\infty} C_{mn} B_n - \sum_{n=0}^{\infty} D_{mn} G_n - \sum_{n=0}^{\infty} (D_{mn} e^{iP_n l_4}) H_n = 0; m = 0, 1, 2, ..., \tag{18g}$$

$$\sum_{n=0}^{\infty} (C_{mn} \hat{K}_n e^{i\hat{K}_n l_3}) A_n - \sum_{n=0}^{\infty} (\hat{K}_n C_{mn}) B_n - \sum_{n=0}^{\infty} (P_n D_{mn}) G_n + \sum_{n=0}^{\infty} (D_{mn} P_n e^{iP_n l_4}) H_n = 0; m = 0, 1, 2, ..., \tag{18h}$$

$$\sum_{n=0}^{\infty}(E_{mn}e^{iP_nl_4})G_n+\sum_{n=0}^{\infty}E_{mn}H_n-\sum_{n=0}^{\infty}F_{mn}M_n-\sum_{n=0}^{\infty}(F_{mn}e^{iS_nl_5})N_n=0; m=0,1,2,..., \tag{18i}$$

$$\sum_{n=0}^{\infty}(E_{mn}P_ne^{iP_nl_4})G_n-\sum_{n=0}^{\infty}P_nE_{mn}H_n-\sum_{n=0}^{\infty}S_nF_{mn}M_n+\sum_{n=0}^{\infty}(S_nF_{mn}e^{iS_nl_5})N_n=0; m=0,1,2,..., \tag{18j}$$

$$\sum_{n=0}^{\infty}F_{mn}e^{iS_nl_5}M_n+\sum_{n=0}^{\infty}F_{mn}N_n-\sum_{n=0}^{\infty}E_{mn}T_n=0; m=0,1,2,..., \tag{18k}$$

$$\sum_{n=0}^{\infty}F_{mn}S_ne^{iS_nl_5}M_n-\sum_{n=0}^{\infty}S_nF_{mn}N_n-\sum_{n=0}^{\infty}P_nE_{mn}T_n=0; m=0,1,2,..., \tag{18l}$$

$$\sum_{n=0}^{\infty}(P_ne^{iP_nl_4})G_{mn}G_n-P_nG_{mn}H_n=0; m=0,1,2,..., \tag{18m}$$

$$\sum_{n=0}^{\infty}P_nG_{mn}T_n=0; m=0,1,2,..., \tag{18n}$$

$$\sum_{n=0}^{\infty}(A_n-e^{iP_nl_3}B_n)P_nH_{mn}=0; m=0,1,2,..., \tag{18o}$$

$$\sum_{n=0}^{\infty}(e^{iP_nl_3}A_n-B_n)P_n\hat{H}_{mn}=0; m=0,1,2,..., \tag{18p}$$

where

$$\epsilon_m = \begin{cases} \hat{N}_m, \text{for } m=0 \\ 0, \text{for } m\neq 0 \end{cases} \tag{19a}$$

$$A_{mn}=\int_0^{h_1} g_n\psi_m\,dy, B_{mn}=\int_0^{h_1}\psi_m\hat{\psi}_n\,dy,\ \text{for m, n = 0,1,2, ...,} \tag{19b}$$

$$C_{mn}=\int_0^{h_3}\psi_m\hat{\psi}_n\,dy,\ \ D_{mn}=\int_0^{h_3}\psi_mZ_n\,dy,\ \text{for m, n = 0,1,2, ...,} \tag{19c}$$

$$E_{mn}=\int_{d_3}^{h_3}Z_n\psi_m\,dy,\ \ F_{mn}=\int_{d_3}^{h_3}X_n\psi_m\,dy,\ \text{for m, n = 0,1,2, ...,} \tag{19d}$$

$$G_{mn} = \int_0^{d_3} Z_n \psi_m \, dy, \text{ for m, n = 0,1,2, ...,} \tag{19e}$$

$$H_{mn} = \int_{h_1}^{h_2} \hat{\psi}_n \psi_m \, dy, \ \hat{H}_{mn} = \int_{h_3}^{h_2} \hat{\psi}_n \psi_m \, dy, \text{ for m, n = 0,1,2, ...} \tag{19f}$$

In equations (18a) to (18p) truncating the series in n, m at N terms, we get an over-determined system (ODS) of $(16N + 16)$ equations with $(12N + 12)$ unknowns of the form

$$\hat{A}X = \hat{b} \tag{20}$$

where $\hat{A}$ is of the size $(16N + 16) \times (12N + 12)$ and X is the column vector of unknown coefficients. The least-square solution can be obtained by solving the normal system $\hat{A}^*\hat{A}X = \hat{A}^*\hat{b}$, where $\hat{A}^*$ denotes transpose conjugate of $\hat{A}$. The unique least-square solution is given by $X = (\hat{A}^*\hat{A})^{-1}(\hat{A}^*\hat{b})$ provided $\hat{A}$ has linearly independent columns which can be possibly choosing the values of the parameters appropriately. The effectiveness of floating porous plate as a breakwater can be studied through wave load which are defined as:
The non-dimensional hydrodynamics force on the floating bridge which are defined as:

$$F_v = |\frac{\omega}{gh_3} \int_d^e \phi_6(x, d_3) \, dx|, \tag{21}$$

$$F_h = |\frac{\omega}{gh_3} \int_0^{d_3} \{\phi_7(e, y) - \phi_5(d, y)\} \, dy|. \tag{22}$$

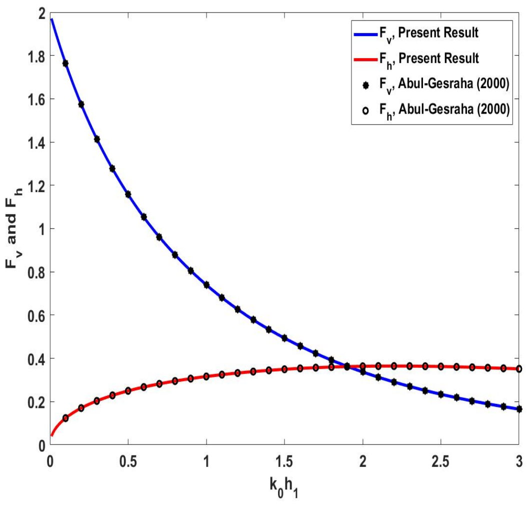


Figure 1: F_v and F_h versus k_0h_1 for fixed $H_2 = H_3 = 1, G = 0, L_1 = L_2 = L_3 = L_4 = 0, L_5 = 2, D_3 = 0.25, \theta_1 = 60^0$

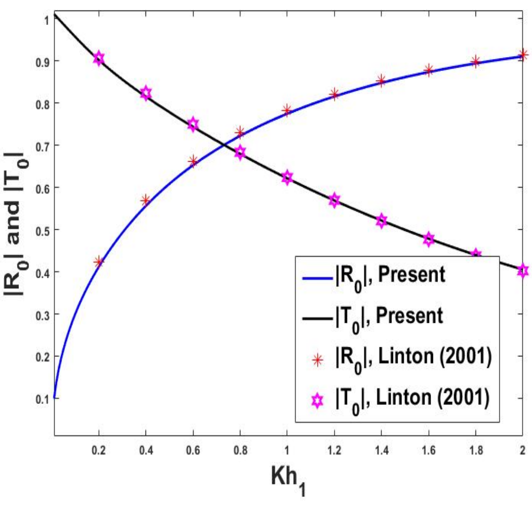


Figure 2: $|R_0|$ and $|T_0|$ versus Kh_1 for fixed $H_2 = H_3 = 1, G = 0, L_1 = 2, L_2 = L_3 = L_4 = L_5 = 0, D_3 = 0, \theta_1 = 0^0$

4 Numerical results

In this section, the hydrodynamic quantities namely, vertical wave force on porous plate, vertical wave force on floating bridge and horizontal wave force on floating bridge, are numerically computed from the equations (20) to (21). The non-dimensional parameters are given as $\theta_1 = 30^0, h_1 = 10m, Kh_1 = \omega^2 h_1/g, H_i = h_i/h_1, L_j = l_j/h_1, for$i=1, 2, 3, $D_3 = d_3/h_1, L_2 = L_4 = 1$, for $j = 1, 2, 3, 4, 5$ otherwise unless mentioned in the paper.

4.1 Validation

For the validation of the computational results obtained in the present work, the present results are compared with the results from the literature for certain limiting cases. In Fig.1 the forces F_v and F_h against wave number k_0h_1 in the absence of porous plate in the uniform finite depth of water are compared with the results of Abul and Gesraha (2000). The graph shows that a good agreement between the present method and the method of Abul and Gesraha is obtained.

In Fig.2, another comparison is made with the particular case involving a rigid dock over a flat bottom (non-appearance of trench $H_2 = H_3 = 1$) and the absence of floating bridge ($L_5 = 0, D_3 = 0$). Here, the result $|R_0|$ and $|T_0|$ shows good agreement with Linton's results.

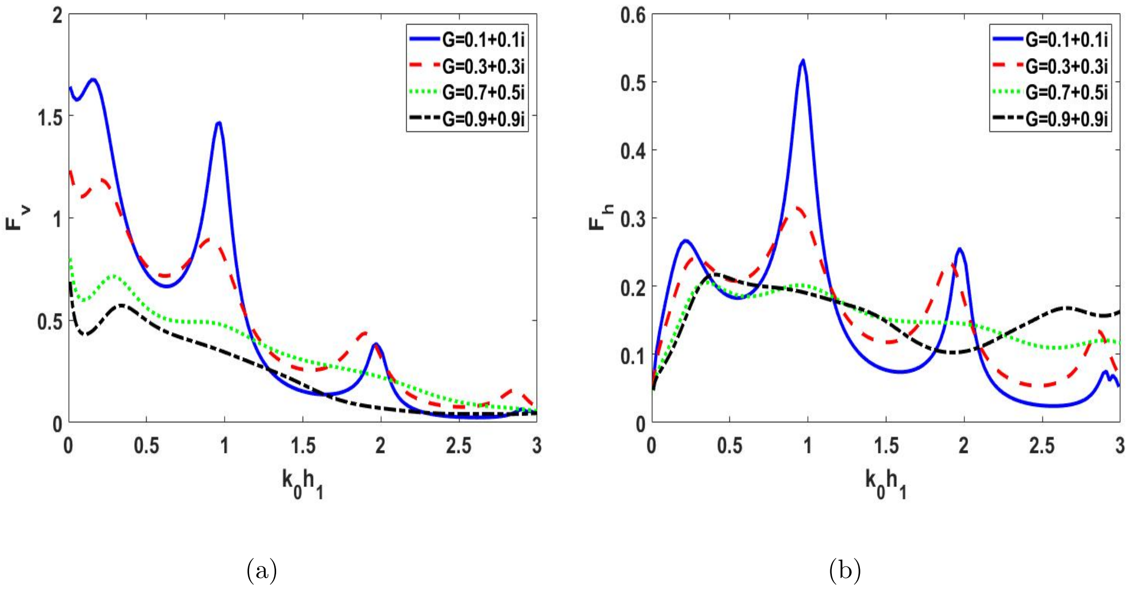
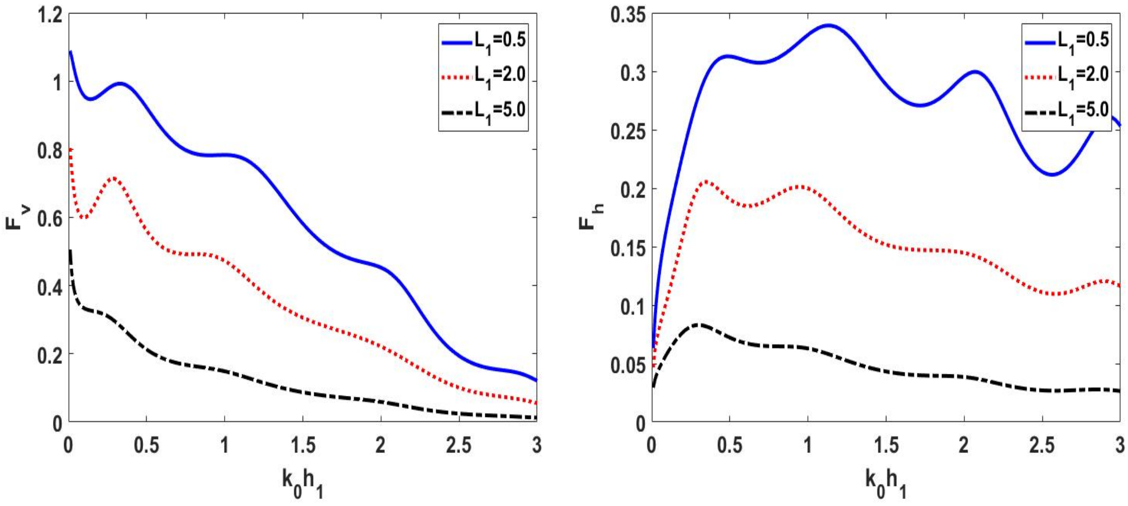


Figure 3: Variations in (a) F_v and (b) F_h versus k_0h_1 for different values of G with fixed values of $H_2 = 1.5, H_3 = 0.8, \theta_1 = 30^0, L_1 = L_3 = L_5 = 2$

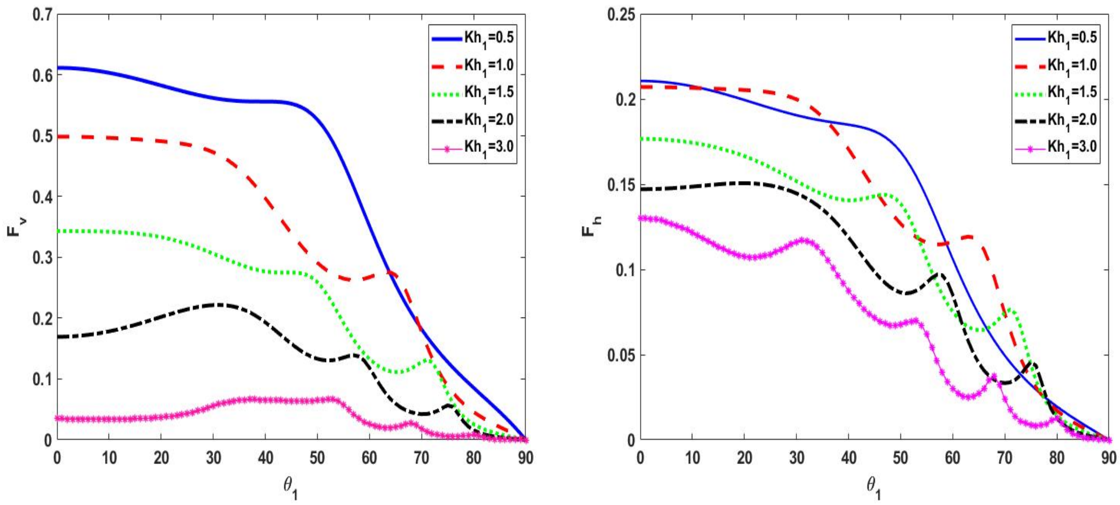
Figure 3 exhibits the variations of vertical forces F_v and horizontal force F_h on a floating bridge for different values of porous effect parameter. It is noticed that all curves show an oscillatory pattern with an optima value. The oscillatory pattern is due to the resonating interaction of the incident wave and reflected wave between the porous plate and the floating bridge. Here, F_v and F_h are reduced as porosity increases because of more incidence wave energy dissipation. It may be noted that the vertical force is higher than the horizontal one due to the impact of incident waves on the floating bridge.

Figure 4 demonstrates the variations of the wave forces as a function of wave numbers for different lengths of the porous plate. It is perceived that the oscillations in the curves with optima exist for smaller values L_1 and when the length increases, both forces decrease on the bridge because of more energy dissipation. The minimum force is observed when the length of



(a) (b)

Figure 4: Variations in (a) F_v and (b) F_h versus k_0h_1 for different values of L_1 with fixed values of $H_2 = 1.5, H_3 = 0.8, \theta_1 = 30^0, G = 0.7 + 0.5i, L_3 = L_5 = 2$



(a) (b)

Figure 5: Variations in (a) F_v and (b) F_h versus θ_1 for different values of Kh_1 with fixed values of $H_2 = 1.5, H_3 = 0.8, G = 0.7 + 0.5i, L_1 = L_3 = L_5 = 2$

the porous plate is large ($L_1 = 5$) with a non-oscillatory pattern. This phenomenon of optima in wave reflection is due to the constructive and/or destructive interference of incident and reflected waves within the structures. In the absence of a porous plate, maximum force is experienced by the floating bridge, but the force decreases in the presence of the porous plate.

The wave forces against incident angle θ_1 are examined in Fig.5 for different values of Kh_1. With the increase in the angle of wave incidence, wave forces decrease up to zero. The reason is that for the higher value of θ_1, more reflection is possible, resulting less forces. Here, the results also reveal that the horizontal force reduces with a higher rate as compared to the vertical force. By comparing the curves for different values of Kh_1, more force is experienced by the floating bridge for smaller values of Kh_1 in the range $0 \leq \theta_1 \leq 70^0$, and after $\theta_1 > 70^0$ (Called as a critical angle) it produces almost same force.

5 Conclusions

The present study investigates the performance of horizontal porous plate over trench to diminish wave force on floating bridge. Based on the eigenfunction expansion method, an over-determined system of linear algebraic equations is obtained which is solved by using least-square method. The numerical results for vertical force and horizontal force are plotted though different graphs for various parameters such as the porosity of plate, length of plate and angle of incidence. The major conclusions are as follows: (i) Increasing the plate porosity it reduces the wave force on the floating bridge. (ii) Increasing the porous plate's length dissipates more incident wave energy and hence reduces the wave forces on the floating bridge. (iii) After the critical angle, the force for all values of Kh_1 is is almost same and reduced up to a minimum value of zero.

Thus, a suitable choice of the values for length of plate, porosity and incidence wave angle can be helpful in the construction of effective breakwaters involving a horizontal porous plate which will create a calm zone in the lee side region.

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