Stochastic RBFN-based reliability estimation of variable fiber spacing composite plates under thermal loading

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ABSTRACT

Stochastic reliability analysis is performed in the present work for variable fiber spacing composite (VFSC) laminates. The finite element model here is developed with third-order shear deformation theory (TSDT) to estimate buckling performance under thermal loading. The reliability analysis is performed with the first-order reliability method (FORM) and with the radial basis function network (RBFN) model as well. Further RBFN-based surrogate model is utilized for stochastic analysis, which is highly efficient and possesses a close match with traditionally used Monte Carlo simulation (MCS). The sensitivity of each input parameter is investigated, out of which the thermal expansion coefficient of the matrix (α_m) is found to be the most sensitive one.

Keywords: Stochastic thermal buckling, variable fiber spacing composite (VFSC), First-order reliability method (FORM), Radial basis function network (RBFN), Monte Carlo simulation (MCS).

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1. INTRODUCTION

Composites are a widely used material in all engineering domains, and the aerospace industry is no different. Even though the composite material is itself an improvement over traditional materials, regulating fiber distributions may still improve the performance. The VFSC is one of those improvements where the stiffness of the composite varied with given functions. Leissa and Martin [1] [2] improved the free vibration and buckling response with the use of a single-layered VFSC plate. Shiau and Chue [3] showed the minimization of free-edge inter-laminar stress is possible with VFSC. Shiau and Lee [4] took higher-order elements into consideration to analyze the effect of fiber spacing over stress concentration in a laminated composite plate with a hole. The fiber spacing variation concept is also regarded in reinforced concrete (RC) shear walls, reflecting higher strength [5]. Sharma et al. [6] utilized VFSC to improve aeroelastic performance in a dynamic environment.

In a high-flight dynamic environment, thermal buckling could be one of the causes of failure, as aerodynamic heating takes place. Noor and Burton [7] performed a 3-dimensional thermal buckling analysis in a study. Kabir et al. [8] considered shear locking while investigating buckling performance due to thermal loading. Manickam et al. [9] incorporated uniform and non-uniform temperature distribution over varying stiffness composites. Zhao et al. [10] performed a thermal buckling investigation over variable angle tow (VAT) laminates and showed improvement over traditional composites. Sharma et al. [11] performed a stochastic thermal buckling analysis over VAT laminates.

Unlike conventional materials, composite manufacturing is a complex process, so there are more chances for the material properties to be uncertain. An investigation of low-velocity impact response is done by Karsh et al. [12]. In another study, Karsh et al. [13] introduced stochasticity on pre-twisted functionally graded (FG) plates. With given uncertainty to input variables, Koduru and Haukaas [14] presented the need for reliability estimation with advanced finite element models. Design improvements with the utilization of various reliability assessment techniques are performed by d'Ippolito et al. [15]. Kumar et al. [16] used physics-based FORM to investigate the failure probability for the flutter of the aircraft wing. Borello et al. [17] took both isotropic and composite structures to investigate flutter reliability with the FORM and second-order reliability method (SORM). Allen and Maute [18] proposed reliability-based design optimization of aeroelastic structures utilizing FORM.

In the present work, a VFSC plate with three fiber distribution types (DT) is considered for different boundary conditions, and then material uncertainty is introduced to thermal buckling to estimate reliability, while the sensitivity of each uncertain input parameter is taken as the matter of concern as well.

2. FORMULATION

2.1 VFSC formulation

Based on the rule of mixture, the material properties of VFSC are defined [2]. **Figure 1** presents the VFSC plate, where the fiber distribution is presented to be varied in the ζ - direction (orthogonal to the fiber direction). Functions to distribute fiber volume are given below.

$$DT-1{:}V_f=\left(1-\zeta^2\right)^3$$
$$DT-2{:}V_f=\left(1-\zeta^2\right)^2$$
$$DT-3{:}V_f=\left(1-\zeta^2\right)$$

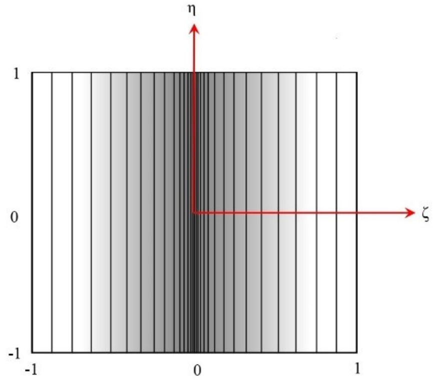

Figure 1: VFSC plate fiber distribution

2.2 Thermal Buckling

The displacement field for TSDT may be approximated as [19] [20],

$$u(x,y,z)=u_0(x,y)+z\theta_x(x,y)+z^2u_0^*(x,y)+z^3\theta_x^*(x,y)$$
$$v(x,y,z)=v_0(x,y)+z\theta_y(x,y)+z^2v_0^*(x,y)+z^3\theta_y^*(x,y)$$
$$w(x,y,z)=w_0(x,y)$$

The above equation is further utilized in the below strain energy equation,

$$\Pi=\frac{1}{2}\int_v\{\varepsilon-\varepsilon_t\}^T\{\sigma\}dv$$

As buckling is an eigen value problem, the final equation comes out to be as follows,

$$\left[[K]+\lambda[K_g^{\delta T}]\right]\{q\}=\{0\}$$

And the critical buckling temperature for the plate is related to eigen value as,

$$T_c=\lambda\,\delta T$$

2.3 First-order reliability method (FORM)

The present work utilized response surface (RS) based FORM. The quadratic equation considered for RS is given as [21]:

$$y(\chi)=\varphi_0+\sum_{i=1}^{m}\varphi_i\chi_i+\sum_{i=1}^{m}\sum_{j=1}^{m}\varphi_{ij}\chi_i\chi_j+\varpi$$

Where, χ_i are the independent input parameters. The limit state function is given as follows:

$$g(x) = f(x) - T_c^m$$

The reliability index φ gets updated iteratively until the convergence to get the final reliability parameter, and the failure probability is given by the equation:

$$P_f = \phi(-\varphi)$$

2.4 Radial basis function network (RBFN)

Modern engineering applications utilize surrogate models to seek optimal solutions in lesser time and RBFN used in current work is from one of those models. The model built by RBFN is,

$$S(\iota) = \sum_{j=1}^{p} \omega_j h\left(\| \iota - \iota_j \|\right)$$

Where, ω_j are the weighted coefficients of j^{th} sample, h is radial basis function, and the Euclidean distance of points to centre is given by $\| \iota - \iota_j \|$. Based on several predefined radial basis functions, the algorithm generates a surrogate model and predicts the output [22]. The estimation of failure probability is done by the equation:

$$P_f = \frac{1}{N} \sum_{i=1}^{N} F\left(g(h^i) < 0\right)$$

3. RESULTS AND DISCUSSION

3.1 Verification study

The verification of the VFSC laminates is presented in **Table 1** and **Table 2**, which indicates the closeness of the current code results with respect to various literature for non-dimensional fundamental frequency and critical buckling temperature, respectively.

Table 1: Non-dimensional fundamental frequency verification table

Lamination sequence	DT	Fundamental natural frequency	
		Kuo [23]	Present FEM
$[0/90/0/90]_s$	$(1-\zeta^2)^3$	11.370	11.364
	$(1-\zeta^2)^2$	12.040	12.022
	$(1-\zeta^2)$	13.110	13.074

Table 2: Critical buckling temperature (°C) verification table for simply supported uniformly distributed fiber composites

Results	Lamination sequence	
	$[0/45/-45/90]_s$	$[0/90/90/0]_s$
Shiau et al. [24]	13.752	12.261
Zhao et al. [10]	13.650	12.250
Present FEM	13.752	12.259

3.2 Thermal buckling response

The material property is taken from reference [1]. The critical buckling temperature for all three DT and boundary conditions is there in **Table 3**, where S stands for simply-supported and C for clamped edge conditions. The maximum value of critical buckling temperature is observed for DT-3 $(1-\zeta^2)$ and C-C-C-C boundary condition. The study here is performed for cross ply laminates.

Table 3: Critical buckling temperature (°C) of VFSC for various conditions

Lamination sequence	DT	Boundary conditions		
		S-S-S-S	C-S-C-S	C-C-C-C
$[0/90/0/90]_s$	$(1-\zeta^2)^3$	15.441	35.743	75.089
	$(1-\zeta^2)^2$	19.115	43.207	88.444
	$(1-\zeta^2)$	25.965	56.827	109.941

3.3 Reliability analysis

Estimation of reliability is performed for cross-ply laminates with two methods, namely the FORM and RBFN-based surrogate model. Three DT are considered for the analysis of reliability. The CDF curve for 5%,10% stochasticity, or the coefficient of variance (COV), is presented in **Figure 2-7**. The visible variation observed with both methods is significantly less. Afterward, the RBFN is considered for reliability estimation (**Figure 8-10**). The failure probability range is observed to be proportional to the COV, where with 5% COV, the range of probability of failure is recorded to be less in comparison to 10% COV.

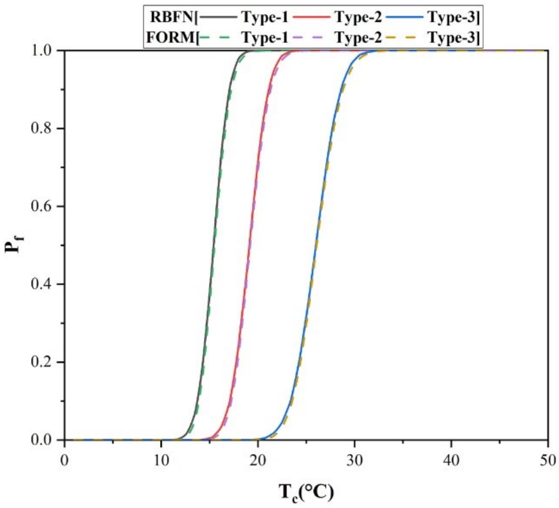

Figure 2: Failure probability at 5% COV, S-S-S-S

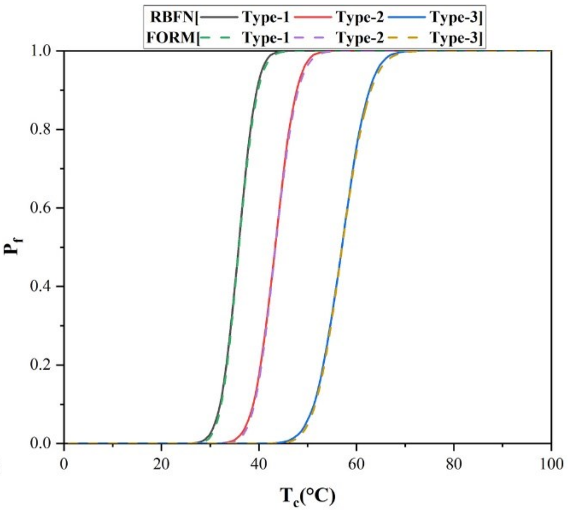

Figure 3: Failure probability at 5% COV, C-S-C-S

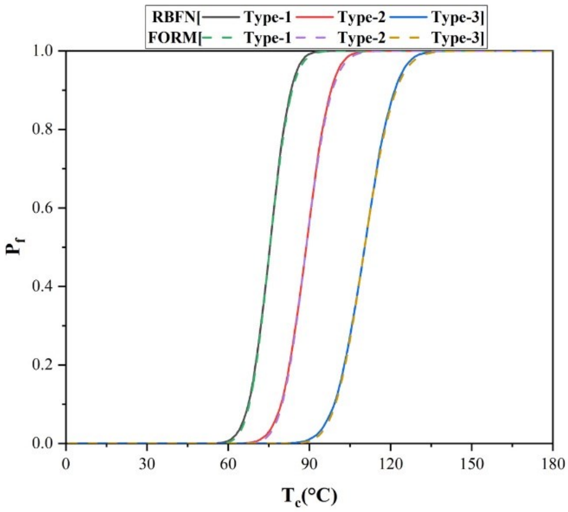

Figure 4: Failure probability at 5% COV, C-C-C-C

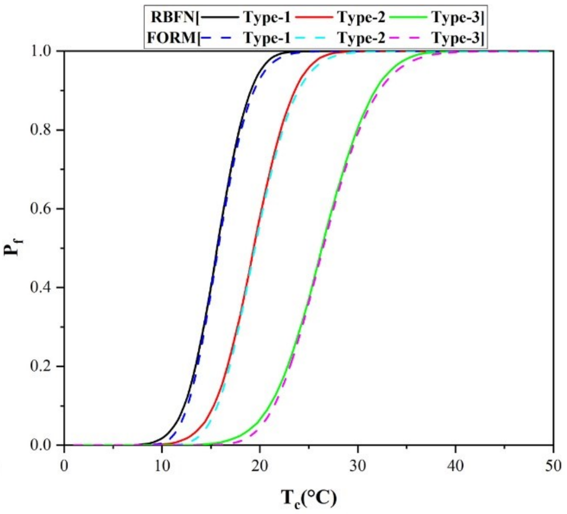

Figure 5: Failure probability at 10% COV, S-S-S-S

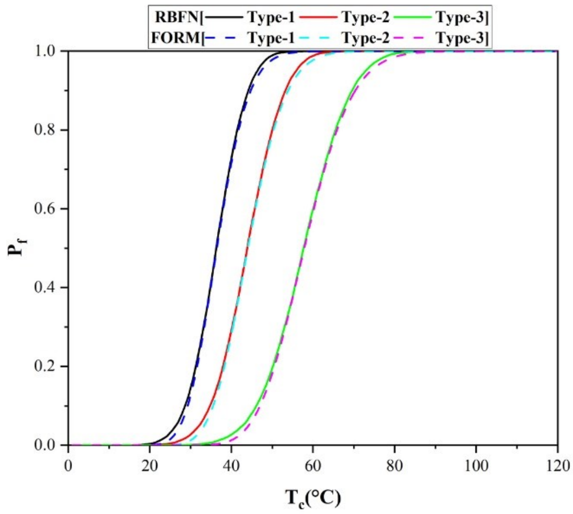

Figure 6: Failure probability at 10% COV, C-S-C-S

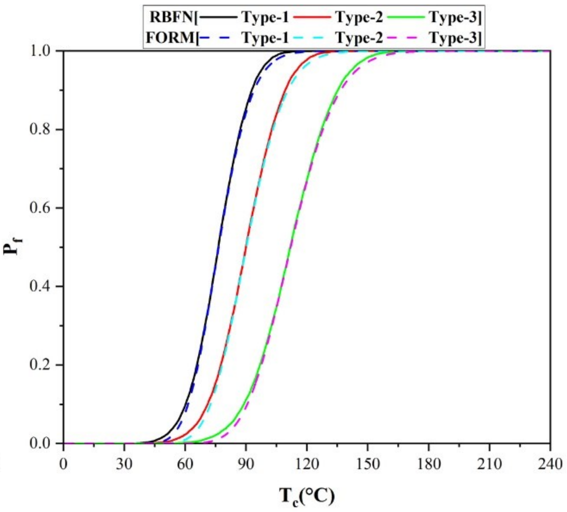

Figure 7: Failure probability at 10% COV, C-C-C-C

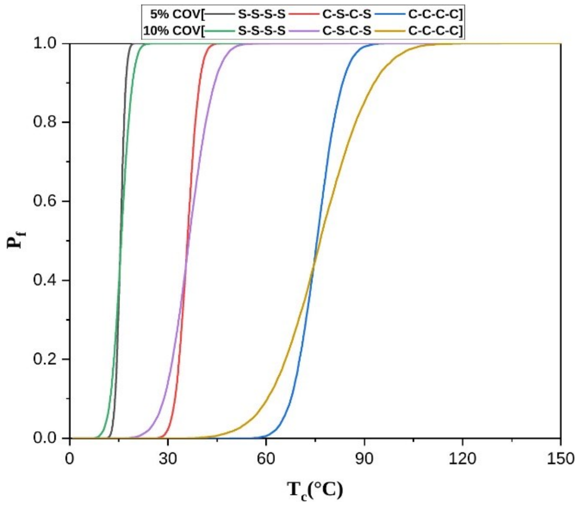

Figure 8: RBFN predicted failure probability, DT-1

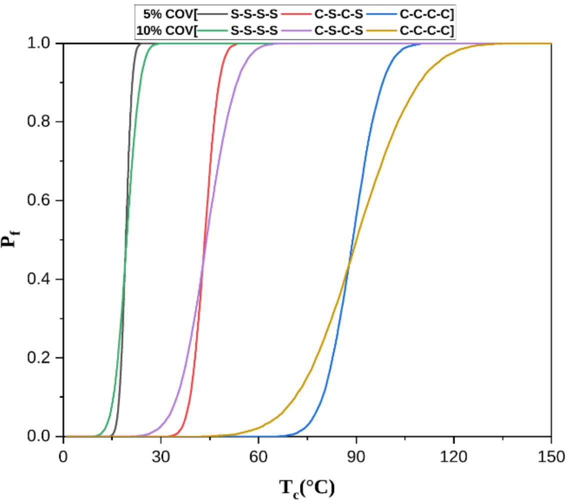

Figure 9: RBFN predicted failure probability, DT-2

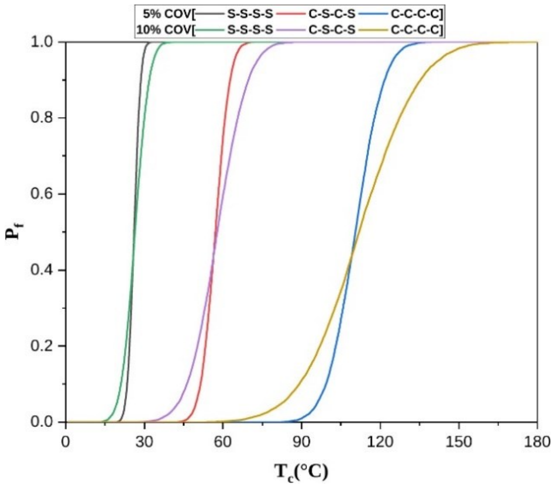

Figure 10: RBFN predicted failure probability, DT-3

3.4 Stochastic analysis

The RBFN-based surrogate model is utilized for stochastic response prediction and converges very well with the MCS method (**Figure 11**). Further, the influence of material (M), thermal (T) and combined (C) properties on output response are estimated individually. The linear relation observed between stochasticity of input and output. The results indicate that the dominance of the combined property effect is the most, followed by the material and thermal properties (**Figure 12-14**).

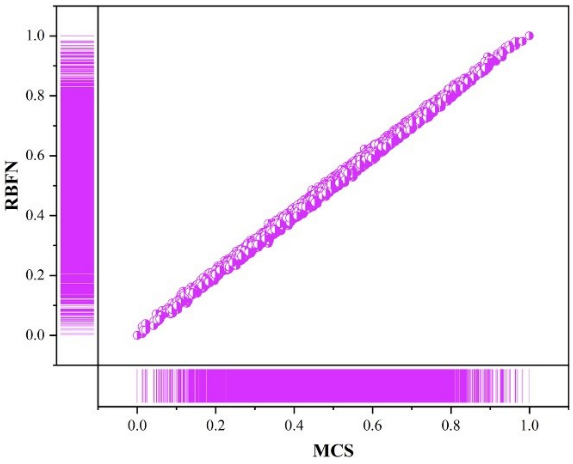

Figure 11: Normalized Scatter plot

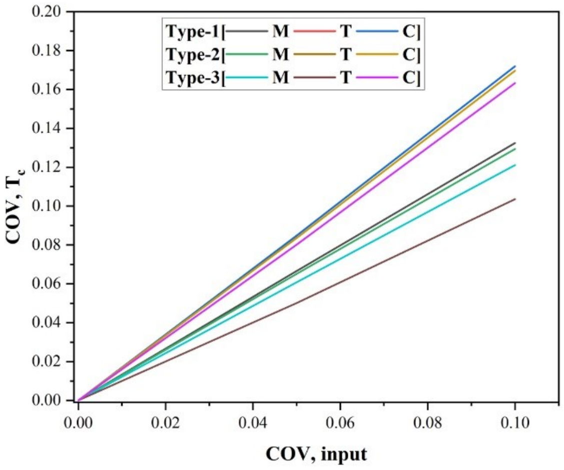

Figure 12: Input property influence on output, S-S-S-S

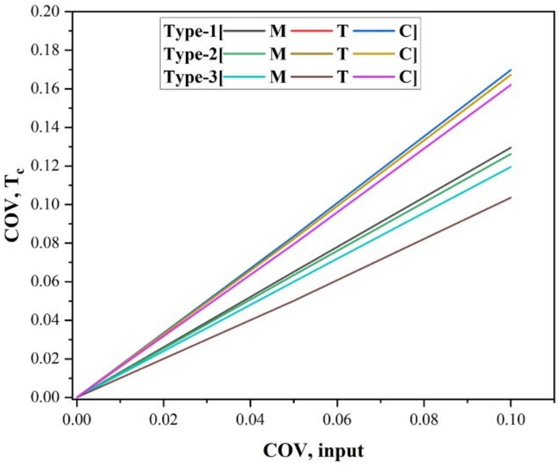

Figure 13: Input property influence on output, C-S-C-S

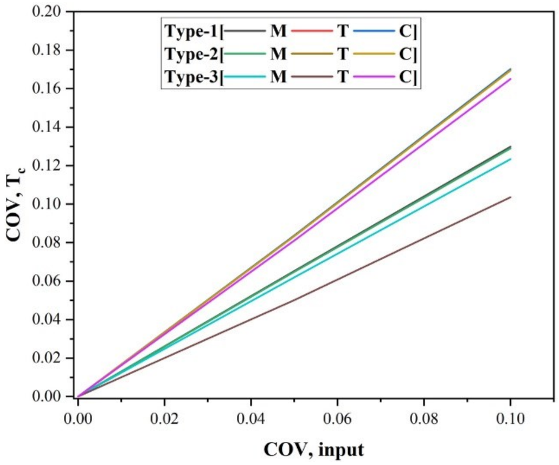

Figure 14: Input property influence on output, C-C-C-C

3.5 Sensitivity analysis

The sensitivity estimation here is performed for cross-ply laminates for various cases. Thermal expansion coefficient of matrix α_m is found to be the most sensitive one, followed by the modulus of elasticity of matrix E_m, and elastic modulus of fiber E_f. The same can be viewed in **Figure 15-17**. In all three edge conditions, almost similar observations recorded.

4. CONCLUSIONS

The following conclusions are drawn from this research work:

- The maximum critical buckling temperature observed for DT-3, C-C-C-C (≈110°C).
- The failure probability (P_f) is adequately revealed by the CDF curves, where the range of probability of failure is observed to be more with higher COV.
- RBFN is more efficient in predicting results with fewer samples than MCS.
- The sensitivity observation recorded is almost similar for all the boundary conditions tested.

- The thermal expansion coefficient of matrix (α_m) is found to be the most sensitive material parameter.
- Input-to-output stochasticity possesses linear relation with the highest influence of combined property stochasticity.

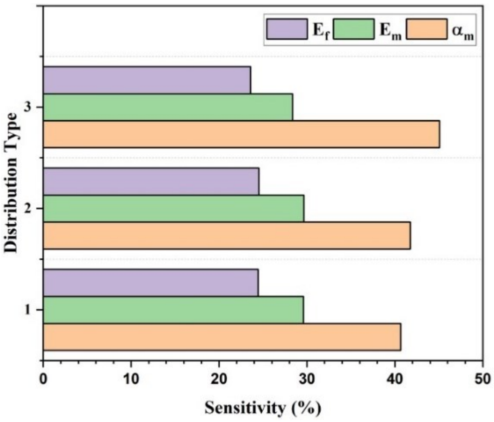

Figure 15: Sensitivity S-S-S-S

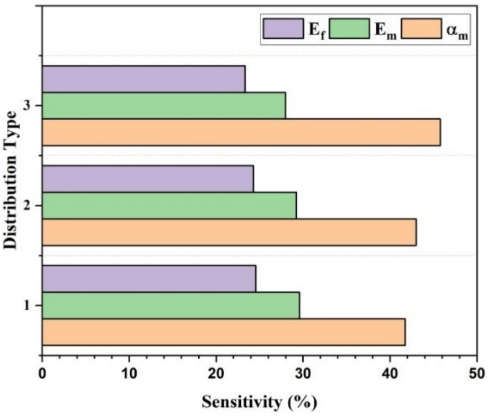

Figure 16: Sensitivity C-S-C-S

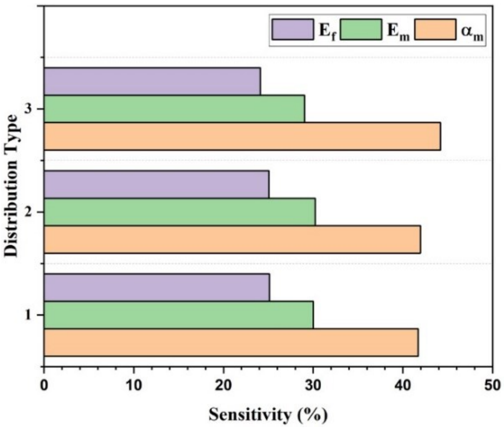

Figure 17: Sensitivity C-C-C-C

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