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Analysis of Surface effect on Shear wave propagation at cylindrical bi-layer with imperfect interface

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Abstract- A theoretical approach is taken into consideration to investigate the surface effect on the propagation behaviour of shear wave in a functionally graded piezoelectric medium (FGPM) over a Fiber Reinforced Composite (FRC) cylindrical model. The interface has been considered that there is a slip between two mediums. The dispersion equation for the shear wave has been derived analytically in the closed form. The effects of the functional gradient parameter, radii ratio, wave number, and surface effect were demonstrated through numerical simulation. The present problem is an important foundation in engineering science for evaluating the interfacial properties, such as non-destructive testing, structural health monitoring, etc.

Literature survey- Surface effect[1], [2] has a significant impact on layer structure at the nano and micro sizes due to the large surface-to-bulk volume which is a main source of these unusual behaviours. Also, nowadays piezoelectric material [3], [4] has numerous applications in the field of design of a variety of devices, including transducers, sensors and generators at the nanoscale. On the other hand, FRC[5], [6] plays important role in construction, aviation, space and sports field. In this article the surface effect has been considered in functionally gradered PE layer overlying FRC layer structure. Till now, no theoretical study has been done in cylindrical layer medium with surface effect, which gives the novelty of the present work. Also, the presence of the inhomogeneity parameter and loose bonding interface[7] makes this model more realistic. The present problem is an important foundation in engineering science for evaluating the interfacial properties, such as non-destructive testing, structural health monitoring, etc.

Basic equation of the surface piezoelectricity-

For the surface layer of the piezoelectric plate, the constitutive equations proposed by Huang and Yu[8] can be written as

$$T^s_{ab} = T^0_{ab} + c^s_{abcd} S_{cd} - e^s_{abk} E_k \tag{1}$$

$$D^s_a = D^0_a + e^s_{acd} S_{cd} + \varepsilon^s_{ak} E_k \tag{2}$$

where the surface stresses and electric displacements can be defined as T^s_{ab} and D^s_a respectively. T^0_{ab} and D^0_a are the surface residual stresses and electric displacements. c^s_{abcd}, e^s_{acd} and ε^s_{ak} are the surface elastic, surface piezoelectric and surface dielectric constants, respectively.

Problem formulation – This model shows the propagation of shear wave in a cylindrical layered structure which is a FRC material imperfectly bonded to FGPE material with surface effect. The width of the FGPM and FRC are $h = (r_2 - r_1)$ and r_1 respectively. It is assumed that the shear wave propagates in the direction of θ and the displacement component (u_1, u_2, u_3) in the direction of (r, θ, z) can be denoted as $(u_1, u_2, u_3) = (0, 0, w)$ (3)

The constitutive relations for piezoelectric material are given by

$$T_{ab} = c_{abcd} S_{cd} - e_{abk} E_k \tag{4}$$

$$D_a = e_{acd} S_{cd} + \varepsilon_{ak} E_k \tag{5}$$

where c_{abcd} is elastic constants, e_{abk} is piezoelectric constants and ε_{ak} is dielectric constants respectively. Also, T_{ab} and S_{cd} are denoted by stress and strain tensor respectively. The electric displacement and intensity of the electric field are denoted by D_a and E_k respectively. The form of heterogeneous of all material parameters is defined as $c_{44} = c'_{44}\left({r}/{r_2} \right)^l$, $e_{15} = e'_{15}\left({r}/{r_2} \right)^l$, $\varepsilon_{11} = \varepsilon'_{11}\left({r}/{r_2} \right)^l$, $\rho = \rho'\left({r}/{r_2} \right)^l$. Solution of FGPM obtained in teams of Bessel's function as

$$\omega' = \left[C_1 J_P\left(\frac{\omega r}{\beta_1} \right) + C_2 Y_P\left(\frac{\omega r}{\beta_1} \right) \right] r^{-\frac{l}{2}} \cos(n\theta - \omega t) \tag{6}$$

$$\phi' = \left[C_3 r^{-P} + C_4 r^P + \frac{e'_{15}}{\varepsilon'_{11}} \left(C_1 J_P\left(\frac{\omega r}{\beta_1} \right) + C_2 Y_P\left(\frac{\omega r}{\beta_1} \right) \right) \right] r^{-\frac{l}{2}} \cos(n\theta - \omega t) \tag{7}$$

Solution for FRC

$$\omega = \omega(r)e^{i(n\theta - \omega t)} \tag{8}$$

Boundary conditions-

A. Condition for loosely bonding at interface i.e., $\tau_{rz}^1 = k_1(u_z^1 - u_z^2)$ at $r = r_1$ (9)

B. Traction is continuous at common interface, i.e., $\tau_{rz}^1 = \tau_{rz}^2$ at $r = r_1$ (10)

C. For piezoelectric short case, $\phi = 0$ at $r = r_1$ and $\phi = 0$ at $r = r_2$ (11)

D. Condition for surface effect at upper surface i.e., $\tau_{rz}^1 = \frac{\mu_s}{r_2^2} u_{z,\theta\theta} - \rho_s \ddot{u}_z$ at $r = r_2$ (12)

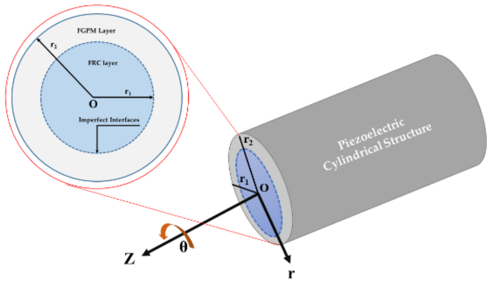

Fig 1: Geometry of considered model

Dispersion equation-Using Eqs. (6)-(7) and Eq. (8) in the boundary conditions Eqs. (9)-(12), a system of five linear homogeneous eqs. is obtained. The dispersion equation can be written as

$$c_{ij} = 0,\ i, j = 1,2,3,4,5 \tag{13}$$

Significant conclusions – The dispersion curve is plotted using Eq. (13) for piezoelectric short cases. The influence of imperfect interface, functionally graded parameter, surface effect, ratio of thickness, piezoelectric parameters on the propagation of shear wave has been studied. Based on present study some outcomes can be evaluate,

A. The phase velocity decreases as the value of imperfect parameter increases.
B. As heterogeneity grows, the intensity of rigidity and density increase in the medium, which causes an unstable nature in phase velocity.
C. The presence of also has a notable effect on the phase velocity of share wave.

Keywords- Functionally graded parameter; Piezoelectric material, Fiber-reinforced composite, Imperfect interface; Surface effect.

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