PAPER FOR THE YOUNG SCIENTIST AWARD

Weakly Nonlinear Analysis of Anisotropic Thermal Diffusivity in Rotating Magnetoconvection

Krishnendu Nayak
Department of Mathematics, National Institute of Technology Warangal, Telangana State, India
Email: kn712156@student.nitw.ac.in

ABSTRACT

The influence of anisotropic thermal diffusive coefficient on the stability of a horizontal fluid planar layer rotating about its vertical axis and permeated by the horizontal homogeneous magnetic field is studied. The linear stability analysis is performed using the separable solutions in the form of horizontal rolls. The weakly nonlinear behavior of the convective motion in the vicinity of the primary instability threshold is studied using the Landau-Ginzburg equation which has the cubic nonlinearity. This amplitude equation is derived using the multiple scale analysis and modified normal mode method. The linear stability analysis of the amplitude equation is performed to investigate the secondary instabilities, such as Eckhaus instability.

LITERATURE SURVEY

A simple model for many geophysical and astrophysical flows, such as oceanic deep convection and the convective outer layer of the Sun, is found in rotating Rayleigh–Bénard convection (RBC) under the influence of magnetic field. Such problems are suitable for conditions in the Earth's mantle. Roberts and Jones (2000) have studied the stability of the horizontal plane layer rotating about a vertical axis and permeated by a horizontal homogeneous magnetic field using separable solutions. The linear stability analysis of this mathematical model near the onset of convection and the influence of isotropic diffusive coefficients are studied in depth. However, the anisotropic thermal diffusivity effects were not included, although the rising importance of the same in the directional heat transport in industrial and natural processes. The heat transport inside the Earth's core affect strongly due to anisotropic thermal diffusion (Braginsky and Meytlis (1990) and Phillips and Ivers (2000)). Donald and Roberts (2004) investigated the influence of anisotropic thermal diffusive coefficient (κ) in RBC under the influence of magnetic field. This problem is extended by Filippi and Brestenský (2020) by considering anisotropic β-effect in the influence of pure-m anisotropic diffusive coefficient (η) and partial q anisotropic diffusive coefficients (η, κ). Further the linear stability analysis of such problem has been studied by Jones and Roberts (2000); Donald and Roberts (2004); Šoltis and Brestenský (2010) and Filippi et. al. (2019) near the onset of convection. More results can be unearthed if the weakly nonlinear analysis is performed on the RMC along with oscillatory convection. Hence an attempt is made in the present study to analyze this established problem, near the convection, using the normal mode method related to the linear stability and the multiple scale analysis related to weakly nonlinear analysis. The linear stability analysis of the derived cubic nonlinear amplitude equation is performed to investigate the secondary instabilities, such as, Eckhaus instability.

PROBLEM STATEMENT

Two horizontal infinite plane layers filled with electrically conducting fluid kept in the homogeneous horizontal magnetic field that and rotating about its vertical axis is considered. The layers are heated from below and cooled from above. As in various settings, most notably in geophysical applications, material property κ is assumed to anisotropic and the corresponding perturbed governing equations are considered as below:

$$R_0\left[\frac{\partial \vec{u}}{\partial t}+\left(\vec{u}\cdot\vec{\nabla}\right)\vec{u}\right]+\hat{z}\times\vec{u}=-\vec{\nabla}P+\Lambda\left(\vec{b}\cdot\vec{\nabla}\right)\vec{b}+\Lambda\frac{\partial \vec{b}}{\partial y}+R\theta\hat{z}+E_z\nabla^2\vec{u}\,,$$

$$\frac{\partial \vec{\mathrm{b}}}{\partial t}=\vec{\nabla}\times\left(\vec{u}\times\vec{b}\right)+\vec{\nabla}\times\left(\vec{u}\times\hat{y}\right)+\nabla^2\vec{b},\ \frac{1}{q_z}\frac{\partial\theta}{\partial t}+\left(\vec{u}\cdot\vec{\nabla}\right)\theta=\nabla_\alpha^2\theta,\ \vec{\nabla}\cdot\vec{u}=0,\ \vec{\nabla}\cdot\vec{b}=0\,.$$

where the notations are explained in Šoltis and Brestenský (2010) and $R_0=$ Modified Rossby Number, $\Lambda=$Elsasser Number, $E_z=$Ekman Number, $R=$Modified Rayleigh Number and $q_z=$Roberts Number. It can be noted that $\nabla_\alpha^2\theta$ in the heat equation takes care about the anisotropic thermal diffusivity.

SOLUTION METHODOLOGY

The above non-dimensional perturbed governing equations have been converted to a single equation by taking the *z*-component of the curl and double curl of the Navier-Stokes equation, the curl of the induction equation and induction equation itself and the equation of heat conduction. The linear stability analysis has been performed using normal mode analysis. The critical R and critical wavenumber have been derived for the case of Oblique rolls, Normal rolls, and Cross rolls with different values of E_z and Λ. The weakly nonlinear stability analysis has been investigated using multiple scale analysis and the Landau-Ginzburg equation is derived. The heat transfer rate has been calculated. The region of Eckhaus instability has been shown for the different values of E_z, Λ, R_o and q_z.

CONCLUSIONS

The influence of anisotropic thermal diffusive coefficient on the marginal and weakly nonlinear stability near the onset of convection of a horizontal planar layer of electrically conducting fluid has been studied. The layer is permeated by a horizontal homogeneous magnetic field and rotates about a vertical axis of rotation. The present study is advanced by using weakly nonlinear analysis with the thermal diffusive anisotropy of stratification anisotropy *(SA)*. It is observed that in the case of parallel rolls for every E_z, the critical R does not depend on the Λ. When the E_z decreases (i.e. low to high rotation) the heat transfer rate (Nu) increases for all R values with fixed values of Λ, R_o and q_z. The Nu decreases with q_z (i.e. high to low thermal diffusion) for all R. When the E_z decreases (i.e. low rotation to high rotation) and q_z increases (i.e. low to high thermal diffusion) the nonlinear coefficient of amplitude equation decreases in the supercritical region for both isotropic and anisotropic cases.

References

1. Roberts, P.H. and Jones, C.A., *"The onset of magnetoconvection at large Pandtl number in a rotating layer. I. Finite magnetic diffusion"*, Geophys. Astrophys. Fluid Dyn. 2000, 92, 289–325.
2. Y. Rameshwar, G. Srnivas, A. Krishna Rao, U. S. Mahabaleshwar and D. Laroze, *"Nonlinear Analysis of Cross Rolls of Electrically Conducting Fluid under an Applied Magnetic Field with Rotation"*, Processes, 2023, (11) 1-30.
3. Filippi, E. and Brestenský, J., *"Anisotropic β-effect in the diffusive processes in the rotating magnetoconvection problem"*, Astronomische Nachrichten, 2020, 969-982.
4. Šoltis, T. and Brestenský, J., Rotating magnetoconvection with anisotropic diffusivities in the Earth's core. Phys. Earth Planet. Inter. 2010, 178, 27–38.
5. Filippi, E., Brestenský, J., and Šoltis, T., *"Effect of anisotropic diffusion on onset of rotating magnetoconvection in plane layer; stationary mode"*, Geophys. Astrophys. Fluid Dyn. 2019, 113, 80.
6. Pesch, W. and Kramer, L., *"Nonlinear analysis of spatial structures in two-dimensional anisotropic pattern forming system"*, Z. Phys. B 63, 1986, 121.
7. Braginsky, S.I. asnd Meytlis., V.P., "*Local turbulence in the Earth's core*", Geophys. Astrophys. Fluid Dyn., 55(2), 1990, 71.
8. Phillips, C.G. and Ivers., D.J., "*Spherical anisotropic diffusion models for the earth's core*", Phys. Earth Planet. Inter., 117(1–4), 2000, 209.
9. Chandrasekhar, S., *"Hydrodynamics and hydromagnetic stability"*, Clarendon Press, Oxford, 1961.
10. Donald J. T., Roberts P. H., *"The effect of Anisotropic heat transport in the Earth's core on the Geodynamo"* Geophys. Astrophys. Fluid Dynam. 2004, 98, 5, 367–384.
11. Braginsky S. I., Meytlis V. P., *"Local turbulence in the Earth's core",* Geophys. Astrophys. Fluid Dynam., 1990, 55, 2, 71–87.
12. Phillips C.G., and Ivers D.J., *"Spherical anisotropic diffusion models for the Earth's core Phys"*, Earth planet. Int., 2000, 117, 209–223.